

结构动力系统的最优控制问题*

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【摘要】 由结构动力系统的微分方程建立起了动力系统控制的数学模型,然后基于计算结构力学与最优控制的模拟关系,用结构力学中的子结构消元法求解了 Riccati 方程,从而确定了动力系统的最优控制律。数值例题说明了文中方法的有效性。

主题词: 结构动力分析 动力系统 最优化控制

分类号: O342

1 数学模型的建立

结构动力系统的微分方程为

$$M(\xi)\ddot{r} + C(\xi)\dot{r} + K(\xi)r = B_0(\xi)u, \quad r(0) = r_0, \quad \dot{r}(0) = \dot{r}_0 \quad (1)$$

式(1)中, r 为节点的位移向量, ξ 为结构参数向量, $u \in R^m$ 为控制向量, $M(\xi)$ 、 $C(\xi)$ 、 $K(\xi)$ 和 $B_0(\xi)$ 分别为具有相应维数的质量阵、阻尼阵、刚度阵和控制影响阵。通常情况下, $M(\xi)$ 为对称正定阵, 而 $K(\xi)$ 为对称半正定阵。将系统(1)引入 n 维状态空间, 这时 $\dot{x} = A(\xi)x + B(\xi)u$, $x(0) = x_0(\xi)$, 其中

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}B_0 \end{bmatrix}, \quad C = \begin{bmatrix} r \\ \dot{r} \end{bmatrix} \quad (2)$$

设计目标是使下列泛函达到最小。

$$J(\xi, u) = q_1 W(\xi) + \frac{1}{2} q_2 \int [x^T Q(\xi)x + u^T R(\xi)u] dt \quad (3)$$

其中, 加权阵 $Q(\xi)$ 和 $R(\xi)$ 为半正定阵和正定阵, $q_1, q_2 > 0$ 为加权值。

从线性二次控制(LQ 控制)理论看, 对于固定的 ξ , $P = P(\xi)$ 为矩阵 Riccati 方程 $A^T P + PA + Q - PBR^{-1}B^T P = 0$ 的解。求出 $P(\xi)$ 后, 系统的最优控制律为^[1] $u^*(\xi) = -R^{-1}(\xi)B^T(\xi)P(\xi)x = -E(\xi)x$, $E(\xi)$ 为反馈增益矩阵。

2 时段混合能的定义及消元公式的建立

定常离散 LQ 控制系统的对偶方程为^[2]

$$x_{k+1} = \Phi x_k + \Gamma u_k, \quad x_0 = \text{给定向量}, \quad \lambda_k = Qx_k + \Phi^T \lambda_{k+1} \quad (4)$$

式中 $k = 0, 1, 2, \dots, k_f$ 为离散的时间步, $\Phi \in R^{n \times n}$ 、 $\Gamma \in R^{n \times m}$ 、 $Q \in R^{n \times n}$ 且对称非负。

对于每个子结构的特性, 在结构力学中总可用能量式来表征。由于结构力学与最优控制的模拟关系^[2], 对于最优控制中的时段, 自然也可定义出它的能量式, 对其特性进行表征。在处

理最优控制问题时,以采用混合能为宜。现给出时段混合能的定义。

定义:设时段为 $(k, k+1)$, x_k 和 λ_{k+1} 为 k 和 $k+1$ 时刻的“位移向量”和“内力向量”,则时段 $(k, k+1)$ 的混合能算式为

$$V(x_k, \lambda_{k+1}) = -1/2x_k^T Q x_k + 1/2\lambda_{k+1}^T G \lambda_{k+1} + \lambda_{k+1}^T \Phi x_k \quad (5)$$

现研究两个连续时段 $(k-1, k)$ 和 $(k, k+1)$ 的合并消元。文献[2]在给出两个时段混合能的表达式后,基于变分原理,推出了时段消元公式

$$Q_c = Q + \Phi^T(I + QG)^{-1}Q\Phi, \quad G_c = G + \Phi G(I + QG)^{-1}\Phi^T, \quad \Phi_c = \Phi(I + QG)^{-1}\Phi \quad (6)$$

并建立了求解离散 LQ 控制 Riccati 方程的 2^N 类解法。式(6)中下标 c 的阵为合并后时段相应的矩阵。

3 连续时间 Riccati 方程的求解

定常连续线性控制系统的动力学方程为 $\dot{x}(t) = Xx(t) + Uu(t)$, $x(t_0) = x_0$ 。

相应的价值泛函为

$$J = \frac{1}{2}x_f^T S_f x_f + \frac{1}{2} \int [x^T(t) Q x(t) + u^T(t) R u(t)] dt \quad (7)$$

引入 Lagrange 乘子向量 $\lambda \in R^n$, 建立扩展的价值泛函

$$J_e = \frac{1}{2}x_f^T S_f x_f + \frac{1}{2} \int \{x^T(t) Q x(t) + u^T(t) R u(t) + \lambda^T [Xx(t) + Uu(t) - \dot{x}(t)]\} dt \quad (8)$$

由极大值原理,可得对偶方程 $\dot{\lambda} = X\lambda - U^T R^{-1} U^T \lambda$, $\lambda = -Qx - X^T \lambda$ 。令 $\lambda = (\lambda_{k+1} - \lambda_k) / \Delta t$, $\lambda = (\lambda_k - \lambda_{k+1}) / \Delta t$ 可得下列离散方程组

$$x_{k+1} = (I + X\Delta t)x_k - U R^{-1} U^T \Delta t \lambda_{k+1}, \quad \lambda_k = Q\Delta t x_k + (I + X^T \Delta t)\lambda_{k+1}$$

现在可用离散 Riccati 方程的求解方法来处理连续时间的 Riccati 方程,在应用式(6)进行迭代时,应注意下列对应关系:此处的 $Q\Delta t$ 对应(6)中的 Q ; $U R^{-1} U^T \Delta t$ 对应 G ; $I + X\Delta t$ 对应 Φ 。

4 数值例题

已知一动力系统,如图 1 所示。 $k_1 = k_2 = k_3 = 1$, $m_1 = m_2 = m_3 = 1$, $C = 0$ 。

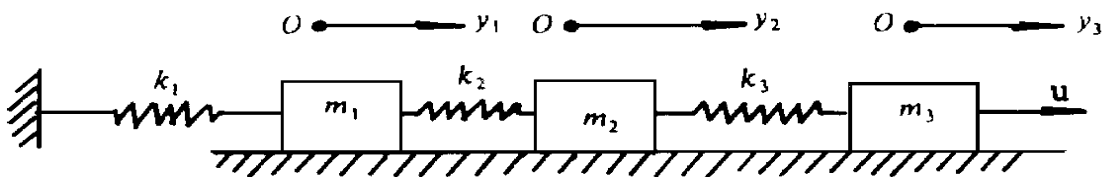


图 1 动力系统示意图

令 $q_1 = q_2 = 1$, $B_0 = \text{diag}(1, 1, 1)$, $R = \text{diag}(1, 1, 1)$, $Q = \text{diag}(1, 1, 2, 2, 3, 3)$ 。系统的刚度阵和质量阵分别为

$$K = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \quad M = \text{diag}(1, 1, 1)$$

由式(2)可求得此时的 A 和 B 。在用消元公式(6)进行迭代时,注意 Q 、 G 、 Φ 的对应关系。经过

迭代求解,可求出 Riccati 方程的解及相应的反馈增益矩阵

$$P = \begin{bmatrix} 885.7471 & & & & & \\ -3.5923 & 923.3380 & & & & \\ 0.0124 & -3.5172 & 920.0421 & & & \\ 41.8912 & 0.0141 & 0.0005 & 4.0949 & & \\ 0.0161 & 42.7748 & 0.0178 & 0.0005 & 4.1393 & \\ 0.0005 & 0.0178 & 42.7988 & 0.0000 & 0.0006 & 4.1400 \end{bmatrix} \quad \text{对 称}$$

$$E = \begin{bmatrix} 41.8912 & 0.0141 & 0.0005 & 4.0949 & 0.0005 & 0.0000 \\ 0.0161 & 42.7748 & 0.0178 & 0.0005 & 4.1393 & 0.0006 \\ 0.0005 & 0.0178 & 42.7988 & 0.0000 & 0.0006 & 4.1400 \end{bmatrix}$$

求得了 E ,就可方便地根据不同时刻的状态得到相应的最优控制律。

当系统为奇异及非线性系统时,文献[3,4]分别给出了相应的算法。

5 结束语

本文的特点是借助于结构力学中的方法,有效地处理了结构动力系统的最优控制问题。最近的研究表明,结构动力学与最优控制中的辛(symplectic)代数有着密切的关系,这样就可基于辛代数学的理论和方法推导出一系列结构动力学的新方法^[5]。

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OPTIMIZATION OF MATERIALS YIELD LIMIT FOR RELIABILITY DESIGN

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ABSTRACT

According to aeroengine structure integrity program, fracture critical components must be analysed with damage tolerance concept. Due to crack propagation produced by material defects, working defects or fatigue defects can make structure fracture. Therefore, the sufficient damage tolerance should be maintained. It is realized by selecting material correctly, controlling stress level, using the anti-fracture structures and reliable inspection. In this paper, the structural reliability concept is applied to studying optimization of materials for maximum reliability under certain stress levels and crack sizes, to investigating variation influences of material strength and stress level on structure reliability.

Key words Reliability design Damage Yield limit

OPTIMAL CONTROL OF STRUCTURAL DYNAMIC SYSTEM

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ABSTRACT

A control mathematical model of the dynamic system is established according to the differential equation obtained from the structural dynamic system. Mixed-energy of time-interval is defined and condensation formulas are given with the help of the variational principle. Further the substructural condensation method in structural mechanics is used for the Riccati equation solution, based on the simulation relation between computational structural mechanics and optimal control. Then, the optimal control of the dynamic system is obtained. The numerical example shows the effectiveness of this method.

Key words Dynamic structural analysis Dynamical systems Optimal control

A $k-\epsilon-k_p-\epsilon_p$ TURBULENCE MODEL FOR GAS-SOLID TWO-PHASE FLOWS AND ITS APPLICATION

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ABSTRACT

Based on $k-\epsilon$ two-equation turbulence model of gaseous phase, the $k-\epsilon-k_p-\epsilon_p$ two-phase turbulence model is presented, which includes the turbulent kinematic energy equation of particulate phase and the dissipation rate equation of that energy. The numerical computations are made for the turbulent gas-solid two-phase flows in two vertical straight channels. The mean velocities and turbulence intensities of gaseous phase and particulate phase, which are predicted with the presented turbulence model, are in good agreement with the experimental results. And it is concluded, that the turbulence intensity of particulate phase is often greater than that of gaseous phase. This conclusion is consistent with available LDV experimental results. Since the algebraic model of particles has no ability to describe the characteristics, the presented turbulence model is very successful in predicting the flow characteristics of turbulent gas-solid two-phase flows.

Key words Two-phase systems Coupling Turbulence model